The Case of the Careless Zookeeper

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Overview of Lesson Plan

In this activity students will have the opportunity to collect and explore real data using a box of animal crackers. The students will use their box of animal crackers to determine if there is a relationship between the type of animal (herbivore or carnivore) and the injury status of the animal (damaged or not damaged) using a chi-square test of independence. While performing the chi-square test, students will walk through every step required in a hypothesis test including: setting up the hypotheses, checking the assumptions, calculating the test statistic, finding the p-value, and interpreting the conclusion in the context of the problem.

GAISE Components

This activity follows all four components of statistical problem solving put forth in the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report.* The four components are: formulate a question, design and implement a plan to collect data, analyze the data by measures and graphs, and interpret the results in the context of the original question. This is a GAISE Level C Activity.

Common Core State Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.

Common Core State Standard Grade Level Content (High School)

S-ID. 5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

S-IC. 1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

NCTM Principles and Standards for School Mathematics
Data Analysis and Probability Standards for Grades 9-12

Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them:

- understand the meaning of measurement data and categorical data, of univariate and bivariate data, and of the term variable.

Select and use appropriate statistical methods to analyze data:

- display and discuss bivariate data where at least one variable is categorical.

Prerequisites

Before beginning this activity, students should be practiced in hypothesis testing for univariate categorical data. The students should also have been introduced to chi-square tests as this activity is intended to give students an opportunity to carry out a chi-square test by hand.

Learning Targets

After completing this activity, students will first be able to create a two-way table from raw data. Then, they will know how to determine if two categorical variables are independent of each other using a chi-square test of independence. In the process of completing this activity, students will also learn how to perform every step of the chi-square test.

Time Required

The time required for this activity is roughly 1 class period.

Materials Required

The students will need to bring a pencil and their graphing calculator, preferably a TI-84 PLUS. The teacher will provide students with the Activity Worksheet (pages 9 through 12) and boxes of animal crackers. Students will be put in groups of two so the instructor only needs to provide boxes for half the class.

Instructional Lesson Plan

The GAISE Statistical Problem-Solving Procedure

I. Formulate Question(s)

Begin the activity by explaining the scenario of the careless zookeeper to give students a clear idea of the problem at hand. A new zookeeper must deliver a trainload of animals to another zoo and due to his/her lack of experience the herbivores (plant eaters) were shipped in the same cars as carnivores (meat eaters). During transit, the animals got hungry and when the train arrived at the destination, the zookeeper found that some animals suffered damage from attacking/being

attacked. Each group of students will be given 1 train car (one box of Barnum's Animal Crackers or approximately 20 to 25 animal crackers) and are asked to assess the damage in that particular car. The goal for each group is to answer the following question: is the type of animal (herbivore or carnivore) related to the injury status (no damage or some damage)? The rest of this investigation will lead to discovering the answer.

Note: Depending upon the type of animal cracker used, the teacher may want to assure that sufficient damage is incurred to the animal crackers prior to having the students collect data.

II. Design and Implement a Plan to Collect the Data

Divide the students into pairs and give each pair one box of animal crackers. This box represents 1 train car from the population of all trains. In the Activity Worksheet (page 12) a description and pictures of the carnivorous and herbivorous animals is given. Students will group the crackers into herbivores and carnivores and then assess the damage to each animal. Let the group determine how they will gauge the damage suffered for an animal, but remind them to be consistent and unbiased throughout the process.

The following table contains example data collected from this activity in an introductory statistics course. There were 15 pairs of students in this section.

Table 1. Example class data.

Damaged Carnivore	Not Damaged Carnivore	Damaged Herbivore	Not Damaged Herbivore
3	4	5	11
1	4	2	13
4	3	1	9
1	4	3	12
4	5	6	8
0	5	1	14
2	4	2	12
1	5	2	13
2	5	1	13
1	4	3	12
3	6	2	10
0	3	2	15
8	14	1	8
0	3	2	15
2	3	6	10

To see how students should complete the remainder of the activity, the questions on the Activity Worksheet will be answered in parts III and IV using the first row of the table.

III. Analyze the Data

Once the students have grouped the animals and assessed their injury status, they will need to fill out all 9 cells of the two-way table on the Activity Sheet. Using the first row of example data above, the table would look as follows.

Table 2. Example individual two-way table.

Type of Animal	Injury Status		Row Totals
	Damaged	Not Damaged	
Carnivore	(f) 3	(g) 4	(b) 7
Herbivore	(h) 5	(i) 11	(c) 16
Column Totals	(d) 8	(e) 15	(a) 23

Students are asked to identify certain attributes for the animal type and injury status variables. The animal type variable is categorical, nominal, and is the independent variable. The injury status is categorical, nominal, and is the dependent variable. Typically, the dependent is the column variable. The students should be aware that this analysis requires both variables to be categorical, so neither variable should be given a quantitative attribute.

Questions 3 through 11 have already been answered in the two-way table above. The students simply sum across all rows and all columns to get the respective values. When students reach questions 12 through 15, they need to find the row percentages for animal type. This means that .429 (3/7) is the proportion of carnivores experiencing damage and .571 (4/7) is the proportion of carnivores experiencing no damage. Similarly, .313 (5/16) is the proportion of herbivores with damage and .688 (11/16) is the proportion of herbivores with no damage. Using these 4 proportions, the students should compare .429 to .313 and then .571 to .688 to answer question 16. If these proportions are deemed to be roughly the same, then no evidence exists to show that a relationship exists between type of animal and injury status. This would tell students that the proportion of damaged and not damaged is the same regardless of animal type, so the variables would be deemed independent. If students see that the proportion of damaged herbivores is low and the proportion of damaged carnivores is high, then there may be a relationship between injury status and animal type. To address this relationship statistically, have the students answer questions 17 to 21, which will outline a chi-square test of independence.

Since the students should have some background knowledge on chi-square tests of independence, they should be able to write the hypotheses correctly. The null hypothesis is Ho: *Animal type and injury status are independent* and the alternative hypothesis is Ha: *Animal type and injury status are dependent*. With the hypotheses stated, the students need to determine if the chi-square test is appropriate. The two major assumptions the students

need to check revolve around expected cell counts. The expected cell counts stem from the idea of conditional probabilities and the definition of independence from a probability perspective. In general, to find the expected cell count for any cell, the students need to multiply the row total and column total for a given cell and then divide that by the overall total. So the expected cell counts for the table of example data would be:

Expected Cell Counts	Injury Status		
Type of Animal	Damaged	Not Damaged	Row Totals
Carnivore	(f) $\dfrac{7*8}{23} = 2.435$	(g) $\dfrac{7*15}{23} = 4.565$	(b) 7
Herbivore	(h) $\dfrac{16*8}{23} = 5.565$	(i) $\dfrac{16*15}{23} = 10.435$	(c) 16
Column Totals	(d) 8	(e) 15	(a) 23

All of these expected cell counts need to be greater than 1 and at least 80% of the cell counts must be greater than 5. Clearly, in this example, 50% of the expected cell counts are less than 5 so the assumptions are not met. Because a basic statistics course does not address Fisher's Exact Test, have the students continue with the activity noting that the assumptions aren't met and the results may not be appropriate.

The students will need these expected cell counts to calculate the test statistic by hand. This is rather simple to do because the chi-square test statistic is calculated using the equation,

$$\chi^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}.$$

Applying this equation to the sample data, the test statistic is,

$$\chi^2 = \frac{(3-2.435)^2}{2.435} + \frac{(4-4.565)^2}{4.565} + \frac{(5-5.565)^2}{5.565} + \frac{(11-10.435)^2}{10.435} = .289.$$

On the TI-84 PLUS calculator, students can use the test statistic value to find the corresponding p-value. Select 2nd → DISTR → χ^2cdf(→ ENTER. Within the parentheses, the students need to enter the lower bound, upper bound, degrees of freedom. The lower bound will always be the test statistic due to the shape of the chi-square distribution. For the upper bound, students can enter any very large number such as 10000000. The degrees of freedom are found using the formula $df = (r-1)(c-1)$, where r is the number of rows in the table and c is the number of columns. Note that in a two-by-two table, the degrees of freedom always equal 1. For this

example, the p-value is .591. Since this value is far greater than .05, students should fail to reject the null hypothesis.

IV. Interpret the Results

From the hypothesis test in this example, students should conclude that at the 5% significance level, there is insufficient statistical evidence to indicate that the animal type and injury status are dependent. This could also be interpreted as a lack of a relationship between animal type and injury status.

The last question of the activity tries to get students to think about inference on a population. In the background to the activity, it is clearly stated that a whole train of animals must be moved and each group of students gets one car from that train. Essentially, this is a cluster sample where each car is a cluster and each sample is a census of one cluster. Therefore, students need to realize that with the sample, it is perfectly reasonable to attempt inference because here they are trying to sample just one car of animals to make a decision about the entire train of animals. In a real situation, this is clearly more feasible and efficient than looking at every animal in the train. However, if the students were to pool all of the data into one two-way table, inference cannot be attempted. The data would come from the population so students have all the possible information about the train. That is why this question is important because it reinforces the goal of inference: use sample data to make conclusions about the population.

Assessment

1. The following table contains data collected to test the relationship between sex and the answer to the question, "With whom is it easiest to make friends?" Using an alpha level of .05, test the following hypotheses:

$$Ho: sex\ and\ finding\ friends\ are\ independent$$
$$Ha: sex\ and\ finding\ friends\ are\ dependent$$

	Opposite Sex	Same Sex	No Difference	Total
Female	58	16	63	137
Male	15	13	40	68
Total	73	29	103	205

(a) Calculate the expected cell counts and find the chi-square test statistic.

(b) Use the calculator to find the corresponding p-value.

(c) Make a conclusion from the test and interpret the results.

2. The data below comes from an Oklahoma roadside survey asking male and female drivers under the age of 20 if they consumed any alcohol in the previous two hours.

	Yes	No	Total
Males	77	404	481
Females	16	122	138
Total	93	526	619

Suppose you wanted to run a chi-square test using this data to see if the driver's sex is related to drinking in the last 2 hours. Calculate the expected cell counts and determine if the necessary assumptions are met for a chi-square test.

Answers

1. (a)

Expected Counts	Opposite Sex	Same Sex	No Difference
Female	48.79	19.38	68.83
Male	24.21	9.62	34.17

$$X^2 = 8.515$$

(b) X^2cdf(8.515, 100000, 2) = .014

(c) Since .014 is less than .05, reject Ho. At the 5% significance level, there is sufficient statistical evidence to indicate that sex and the ease of finding friends based on sex are dependent (related).

2.

Expected Counts	Yes	No
Males	72.27	408.73
Females	20.73	117.27

Since all of the expected cell counts are greater than 5, the expected counts are also greater than 1, so the assumptions are met.

Possible Extensions

1. With such a small number of animals within a car, it is likely that the chi-square assumptions will not be met. Therefore, a logical extension of this activity is to use Fisher's Exact Test for the two-way tables when the assumptions are not satisfied.

2. A two-proportion Z-test could be carried out in order to see if there is a significant difference between the proportions of injured carnivores and herbivores.

References

1. *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report*, ASA, Franklin et al., ASA, 2007. http://www.amstat.org/education/gaise/.
2. Assessment questions from: *Mind on Statistics*. Fourth Edition by Utts/Heckard, 2012. Cengage Learning.

The Case of the Careless Zookeeper Activity Worksheet

A zookeeper-in-training was given the responsibility to ship a trainload of animals on loan to another zoo. Not knowing a lot about how to properly ship animals and the need to keep certain animals apart, he placed a menagerie of animals together into a single train car. Unfortunately, the mix contained an assortment of both carnivores (meat eaters) and herbivores (plant eaters). The neophyte zookeeper also forgot to provide an ample supply of food for the animals. Needless to say, the animals got hungry during transit. When the train car was opened on the receiving end, there was quite a sight to behold, with some of the animals exhibiting signs of damage. Some herbivores have suffered damage from being attacked by the carnivores, and some carnivores have suffered damage while being fought off by the herbivores.

Imagine that you are on the receiving end of the animals and part of the triage team. You have been provided with a train car of animals. Refer to the picture on the last page of the activity sheet to discern whether the animal is an herbivore or a carnivore and whether the animal has experienced any damage.

One-by-one, remove an animal from the car and determine whether it is an herbivore or a carnivore. Further, note whether or not the animal has sustained damage while in transit. Record a tally mark in the appropriate cell in the table below. Continue these steps until all animals have been removed from the train car and triaged. You (and your partner if working in teams) will need to create rules for what to do with partial animals and what constitutes damage. Apply these rules consistently across all animals.

Type of Animal	Injury Status		Row Totals
	Damaged	**Not Damaged**	
Carnivore	(f)	(g)	(b)
Herbivore	(h)	(i)	(c)
Column Totals	(d)	(e)	(a)

1. Identify the following three attributes for the variable *Type of Animal*:

Categorical / Quantitative   Independent / Dependent

Nominal / Ordinal / Interval / Ratio

2. Identify the following three attributes for the variable *Injury Status*:

Categorical / Quantitative   Independent / Dependent

Nominal / Ordinal / Interval / Ratio

3. How many total animals were there in your train car (cell "a")? _____

4. How many of the animals in the train car were carnivores (cell "b")? _____

5. How many of the animals in the train car were herbivores (cell "c")? _____

6. How many of the animals exhibited signs of damage (cell "d")? _____

7. How many of the animals did not exhibit signs of damage (cell "e")? _____

8. How many of the carnivores had signs of damage (cell "f")? _____

9. How many of the carnivores did not have signs of damage (cell "g")? _____

10. How many of the herbivores had signs of damage (cell "h")? _____

11. How many of the herbivores did not have signs of damage (cell "i")? _____

12. What proportion of carnivores was damaged while in transit? _____

13. What proportion of carnivores was not damaged while in transit? _____

14. What proportion of herbivores was damaged while in transit? _____

15. What proportion of herbivores was not damaged while in transit? _____

16. Do you think the answers to Questions 12 and 14 provide strong evidence that there is a relationship between the type of animal and injury status? Yes or No? Please Explain

17. Regardless of your answer to 16, set up the necessary hypotheses for a chi-square test for Independence (alpha = .05).

18. Check the assumptions for the test. That is, find the expected cell counts for cells (f), (g), (h), and (i). Are all the expected cell counts greater than 1? Are at least 80% of the cell counts greater than 5? Even if the assumptions aren't met, continue to carry out the test, but note how the assumptions are not met.

19. Use the expected and observed cell counts to calculate the test statistic.

20. Find the p-value using the correct number of degrees of freedom.

21. What conclusion can you make from the p-value found in 20? How would you interpret this in the context of the problem?

22. Suppose all of the class data was compiled into one table and a chi-square test was run. Should this test be carried out? Can you even interpret the results from the test? Why or Why not?